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ON INTEGRALS OF EQUATIONS OF UNSTABLE NEARLY SELF-SIMILAR FLOWS

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One-dimensional or nearly one-dimensional unstable motions of perfect gas are considered. Integrals admitted by the system of equations defining such motions are examined. Since the existence of integrals is associated with some law of conservation, i. e. with some divergent form of presentation of equations of the input system, it is possible by examining all divergent equations of gas-dynamics to derive certain new integrals not previously considered.

1. As the basic system we select the continuity equation, the Euler equation, and the equation of energy conservation

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho v_k}{\partial x_k} = 0 \tag{1.1}$$

$$\frac{\partial \rho v_i}{\partial t} + \frac{\partial}{\partial x_k}(\rho v_i v_k + \delta_{ik} p) = 0 \tag{1.2}$$

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho v_i^2 + \frac{p}{\varkappa - 1}\right) + \frac{\partial}{\partial x_k}\left(\frac{1}{2}\rho v_k v_i^2 + \frac{\varkappa}{\varkappa - 1} v_k p\right) = 0 \tag{1.3}$$

where subscripts i and k assume the values 1, 2, 3, and recurrent subscripts indicate summation.

Below we refer to certain equations as being of divergent form, if their variables appear as derivatives, e. g. Eqs. (1.1)–(1.3). Equations of divergent form are also called laws of conservation.

Instead of Eq. (1.3) it is possible to use the equation of conservation of entropy of a particle

$$\frac{\partial S}{\partial t} + v_k \frac{\partial S}{\partial x_k} = 0, \quad S = \frac{p}{\rho^\varkappa} \tag{1.4}$$

We denote by $A(S)$ an arbitrary function of S and by $A'(S)$ its derivative with respect to S. We multiply Eqs. (1.1) and (1.4) by $A(S)$ and $\rho A\ \ (S)$, respectively, and add the results. We obtain

$$\frac{\partial \rho A(S)}{\partial t} + \frac{\partial}{\partial x_k}(\rho v_k A(S)) = 0 \tag{1.5}$$

Equation (1.5) is of divergent form and contains an arbitrary function of entropy.

Let us transform Eqs. (1.1) and (1.2). For convenience we introduce the following notation:

$$x^\circ = \left\| \begin{matrix} t \\ x_1 \\ x_2 \\ x_3 \end{matrix} \right\| = \left\| \begin{matrix} x_1^\circ \\ x_2^\circ \\ x_3^\circ \\ x_4^\circ \end{matrix} \right\|, \quad v^\circ = \left\| \begin{matrix} 1 \\ v_1 \\ v_2 \\ v_3 \end{matrix} \right\| = \left\| \begin{matrix} v_1^\circ \\ v_2^\circ \\ v_3^\circ \\ v_4^\circ \end{matrix} \right\|, \quad \overset{\circ}{\delta}_{\alpha\beta} = \left\| \begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix} \right\|$$

where subscripts α and β assume the values 1, 2, 3, 4. With the use of these new variables we combine Eqs. (1.1) and (1.2) and write these as

$$\frac{\partial}{\partial x_\beta^\circ} (\rho v_\alpha^\circ v_\beta^\circ + \overset{\circ}{\delta}_{\alpha\beta} p) = 0 \tag{1.6}$$

We introduce one more subscript $\gamma = 1, 2, 3, 4$. Multiplying Eq. (1.6) by x_γ° and subtracting from it Eq. (1.6) written with subscripts β and γ and multiplied by x_α°, we obtain

$$x_\gamma^\circ \frac{\partial}{\partial x_\beta^\circ} (\rho v_\alpha^\circ v_\beta^\circ + \overset{\circ}{\delta}_{\alpha\beta} p) - x_\alpha^\circ \frac{\partial}{\partial x_\beta} (\rho v_\gamma^\circ v_\beta^\circ - \overset{\circ}{\delta}_{\gamma\beta} p) = 0 \tag{1.7}$$

We transform the first term of Eq. (1.7) to

$$\frac{\partial}{\partial x_\beta^\circ} (\rho x_\gamma^\circ v_\alpha^\circ v_\beta^\circ + x_\gamma^\circ \overset{\circ}{\delta}_{\alpha\beta} p) - \rho v_\alpha^\circ v_\gamma^\circ - \overset{\circ}{\delta}_{\gamma\alpha} p$$

The second terms is similarly transformed. Taking into consideration the symmetry of $\delta_{\alpha\gamma}^\circ$ $(\delta_{\alpha\gamma}^\circ = \delta_{\gamma\alpha}^\circ)$, we finally obtain

$$\frac{\partial}{\partial x_\beta^\circ} [\rho v_\beta^\circ (x_\gamma^\circ v_\alpha^\circ - x_\alpha^\circ v_\gamma^\circ) + (\overset{\circ}{\delta}_{\alpha\beta} x_\gamma^\circ - \overset{\circ}{\delta}_{\gamma\beta} x_\alpha^\circ) p] = 0 \tag{1.8}$$

Taking into account that the left-hand part of (1.8) contains an antisymmetric matrix, we conclude that (1.8) generates six divergent form equations. Three of these defined by subscripts $(\alpha = 2,\ \gamma = 3)$, $(\alpha = 2,\ \gamma = 4)$ and $(\alpha = 3,\ \gamma = 4)$ are the equations of conservation of the vector of the motion moment of momentum. The projection of one of these on the x_3-axis in scalar form is defined by

$$\frac{\partial}{\partial t} [\rho (x_2 v_1 - x_1 v_2)] + \frac{\partial}{\partial x_1} [\rho v_1 (x_2 v_1 - x_1 v_2) + x_2 p] + \\ \frac{\partial}{\partial x_2} [\rho v_2 (x_2 v_1 - x_1 v_2) - x_1 p] + \frac{\partial}{\partial x_3} [\rho v_3 (x_2 v_1 - x_1 v_2)] = 0 \tag{1.9}$$

To obtain the remaining three equations it is sufficient to use subscripts $(\alpha = 1,\ \gamma = 2)$, $(\alpha = 1,\ \gamma = 3)$ and $(\alpha = 1,\ \gamma = 4)$. The equation corresponding to the first pair of subscripts written in scalar form is

$$\frac{\partial}{\partial t} [\rho (x_1 - t v_1)] + \frac{\partial}{\partial x_1} [\rho v_1 (x_1 - t v_1) - t p] + \\ \frac{\partial}{\partial x_2} [\rho v_2 (x_1 - t v_1)] + \frac{\partial}{\partial x_3} [\rho v_3 (x_1 - t v_1)] = 0 \tag{1.10}$$

For any arbitrary number $\varkappa$ the number of equations of divergent form reduces to Eqs. (1.1)—(1.3), (1.5) and (1.8); at certain values of $\varkappa$ namely $\varkappa = 3$ for one-dimensional unstable flows with plane waves, $\varkappa = 2$ for two-dimensional unstable flows and $\varkappa = {}^5/_3$ for three-dimensional unstable flows there are two more supplementary equations [5, 6]. The first of these is obtained by multiplying Eq. (1.2) by $-{}^1/_2 x_i$, Eq. (1.3) by t, and adding these

$$\frac{\partial}{\partial t} \left(\frac{1}{2} t \rho v_k^2 + \frac{3}{2} t p - \frac{1}{2} \rho x_k v_k \right) + \tag{1.11}$$

$$\frac{\partial}{\partial x_i}\left[tv_i\left(\frac{\rho v_k^2}{2}+\frac{5}{2}p\right)-\frac{1}{2}px_i-\frac{1}{2}\rho v_i x_k v_k\right]=0$$

The second equation is obtained as a combination of all Eqs. (1.1) — (1.3). By multiplying Eq. (1.1) by $^1/_2 x_i^2$, Eq. (1.2) by $-x_i t$, and Eq. (1.3) by t^2 and adding these, we obtain

$$\frac{\partial}{\partial t}\left[\frac{1}{2}\rho x_k^2 - t\rho x_k v_k + t^2\left(\frac{1}{2}\rho v_k^2+\frac{3}{2}p\right)\right]+ \tag{1.12}$$

$$\frac{\partial}{\partial x_i}\left[\frac{1}{2}\rho v_i x_k^2 - t x_k(\rho v_i v_k+\delta_{ik}p)+t^2 v_i\left(\frac{1}{2}\rho v_k^2+\frac{5}{2}p\right)\right]=0$$

2. Let one of the equations that define the motion of gas be of the divergent form

$$\frac{\partial F}{\partial t}+\frac{\partial \Phi_k}{\partial x_k}=0 \tag{2.1}$$

Let us determine the derivative with respect to time of the integral of quantity F taken over the mobile volume $V(t)$. Using (2.1) and the Ostrogradskii-Gauss formula, we obtain

$$\frac{d}{dt}\iiint\limits_{V(t)} F\,dV = \iint\limits_{\Sigma}(N_k F-\Phi_k)\,d\sigma_k \qquad (dV = dx_1 dx_2 dx_3) \tag{2.2}$$

where the following notation is used: Σ is the surface bounding volume $V(t)$, $d\sigma$ is an oriented element of surface Σ, σ is the vector of a normal to surface Σ, and $\mathbf{N}$ is the displacement velocity of element $d\sigma$.

We pass now to unstable motions. Let the equation which specifies the position $r_2(t, \varphi, \vartheta)$ of the shock wave propagating through the initially cold quiescent gas for considerable times t be of the form

$$r_2 = (bt)^n\left(1+t^{-2m/(\nu+2)}R_2+\cdots\right) \tag{2.3}$$

where b is a dimensional constant; n and m are some positive integers; values of parameter ν are 1, 2, 3 depending on the dimensionality of the problem, and the quantity R_2 can be either constant or a function of angular variables φ and ϑ. For the investigation of perturbations of cylindrically symmetric motions we use the polar system of coordinates r, φ, and for spherically symmetric motions, the spherical system of coordinates r, φ, ϑ. We denote by v_n and v_τ the velocity vector components which are normal and tangent to the surface of strong discontinuity, and by N the shock wave propagation velocity.

If we denote the state of gas immediately ahead of the wave front by 1 and that behind it by 2, the Rankine-Hugoniot conditions assume the form

$$v_{n2}=\frac{2}{\varkappa+1}N, \quad v_{\tau 2}=0, \quad \rho_2=\frac{\varkappa+1}{\varkappa-1}\rho_1, \quad p_2=\frac{2}{\varkappa+1}\rho_1 N^2 \tag{2.4}$$

Let us first consider the case of $R_2 = \mathrm{const} = 1$. It is possible in that case to seek the solution of problem (2.3) of gas motion behind the shock wave in the form of series in decreasing powers of t with coefficients that are functions of the variable $\lambda = r/(bt)^n$. Since only the radial component v_r of the velocity vector is nonzero, hence

$$v_r = \frac{2n}{\varkappa+1}b^n t^{n-1}\left[f(\lambda)+t^{-2m/(\nu+2)}f_m(\lambda)+\cdots\right] \tag{2.5}$$

$$\rho = \frac{\varkappa+1}{\varkappa-1}\rho_1[g(\lambda) + t^{-2m/(\nu+2)}g_m(\lambda) + \cdots]$$
$$p = \frac{2n^2}{\varkappa+1}\rho_1 b^{2n}t^{2(n-1)}[h(\lambda) + t^{-2m/(\nu+2)}h_m(\lambda) + \cdots]$$

Substituting expansions (2.5) into the system of Eqs. (1.1) — (1.3) and collecting terms of like powers of t, we obtain a system of equations for functions of the first (f, g, h) and second (f_m, g_m, h_m) approximations. First approximation functions define self-similar flows whose general method of analysis was formulated by Sedov [1]; the system of ordinary differential equations which defines such flows appears in [7]. Second approximation functions were investigated in [4] where the system defining these is presented.

If we pass to the variable λ, formula (2.3), which determines the shock front position, assumes the form

$$\lambda_2 = 1 + t^{-2m/(\nu+2)} + \ldots \qquad (2.6)$$

Let us take the volume contained between surfaces $\lambda = \lambda_2$ and $\lambda = \text{const}$ as the mobile volume $V(t)$. We denote by F the terms in the form of derivatives with respect to time in any of the divergent form equations (1.1), (1.3), (1.5), (1.11) and (1.12) in the general case, and in (1.2) and (1.10) in the case of flows with plane waves ($\nu = 1$). Then, substituting functions (2.5) into (2.2) and collecting terms of like powers of t, we obtain

$$\iiint\limits_{V(t)} F\,dV = t^q F_1(\lambda, \lambda_2) + t^{q-2m/(\nu+2)}F_2(\lambda, \lambda_2) + \cdots \qquad (2.7)$$

where q depends on n, ν and the form of F; F_1 and F_2 are one-dimensional integrals with limits of integration from λ to λ_2, with F_1 depending on first approximation functions and F_2 on functions of the first and second approximations.

Similar transformations of the right-hand part of (2.2) yield

$$\iint\limits_{\Sigma} (N_k F - \Phi_k)\,d\sigma_k = t^{q-1}Z_1(\lambda, \lambda_2) + t^{q-2m/(\nu+2)-1}Z_2(\lambda, \lambda_2) + \cdots$$

where Z_1 depends on first approximation functions and Z_2 on first and second approximation functions; both Z_1 and Z_2 independent of derivatives or integrals of related functions.

Selecting F, n and ν so as to have $q = 0$, we immediately obtain a finite relationship for first approximation functions, since in (2.7) the terms associated with first approximation functions vanish after differentiation with respect to time. As the result we have

$$Z_1(\lambda, \lambda_2) = 0 \qquad (2.8)$$

Formula (2.8) satisfies boundary conditions (2.4), since the shock wave (2.6) is taken as the boundary of the mobile volume $V(t)$; on the other hand, it is not difficult to obtain (2.8) in the form of the first integral by substituting for λ_2 some other λ_1.

Selecting F from Eqs. (1.1) and (1.3), and determining the corresponding n and ν from the condition $q = 0$, we obtain the integrals of self-similar motions [1], namely, the integrals of mass and energy that exist only for completely determined n and ν. The integral generated by Eq. (1.5) (the adiabatic integral [1]) exists for any n and ν, since condition $q = 0$ can be always satisfied by an appropriate selection of $A(S)$.

In the case of flows with plane waves ($\nu = 1$) Eq. (1.2) generates the momentum integral [1]. Equations (1.10) and (1.12) are unimportant for the derivation of the first approximation functions, since the condition $q = 0$ becomes in that case $n = 0$. Equation (1.9) is applicable to expansions of other forms than (2.5); more will be said about this subsequently. There remains Eq. (1.11) for which the condition $q = 0$ generates $n = 1 / (\nu + 2)$; on the other hand the divergent equation (1.11) itself holds (with the use of ν) for $\varkappa = (\nu + 2) / \nu$. Introducing the arbitrary constant C_1, we write (2.8) in the form of the first integral

$$\begin{gathered}(h + gf^2)\left(f - \frac{\nu+1}{\nu}\lambda\right) + \frac{2}{\nu}hf - \\ \frac{(\nu+2)(\nu+1)}{\nu}\lambda\left[gf\left(f - \frac{\nu+1}{\nu}\lambda\right) + \frac{1}{\nu}h\right] = \frac{C_1}{\lambda^{\nu-1}} \\ \varkappa = (\nu+2)/\nu, \quad h = 1/(\nu+2)\end{gathered} \tag{2.9}$$

Integral (2.9) is of a fairly simple structure. It consists, as Eq. (1.11) that generates it, of two parts, with the first two terms corresponding to the energy integral, and the last to the momentum integral [1] mutiplied by λ. Integral (2.9) complements the set of first integrals of self-similar motions of perfect gas [1], furthermore, since the number of divergent forms is limited to those defined above [6], there are no other first integrals determined by laws of conservation.

Let us consider second approximation functions. We again select F, n, ν and m so that

$$q - 2m / (\nu + 2) = 0 \tag{2.10}$$

which yields the final relationship

$$Z_2(\lambda, \lambda_2) = 0 \tag{2.11}$$

for the second approximation functions. This relationship may be reduced to the form of the first integral by substituting λ_1 for λ_2. As in the case of first approximation functions, Eq. (1.5) generates an integral for any m [2, 3]. The integrals for second approximation functions for Eqs. (1.1) – (1.3) were considered in [4].

Let us first consider Eqs. (1.11) and (1.12). Selecting F in conformity with Eq. (1.11), we obtain that condition (2.10) is satisfied for $m = (\nu + 2)[(\nu + 2)n - 1] / 2$. We introduce the arbitrary constant C_2 and present (2.11) in the form of the first integral

$$\begin{gathered}\lambda(2gff_m + f^2g_m + h_m) - \frac{\nu}{\nu+1}\left[\left(3f^2g + \frac{\nu+2}{\nu}h\right)f_m + \right. \\ \left. f^3g_m + \frac{\nu+2}{\nu}fh_m\right] - \frac{\nu+1}{\nu n}\lambda\left[\lambda(gf_m + fg_m) - \right. \\ \left. \frac{\nu}{\nu+1}\left(2gff_m + f^2g_m + \frac{1}{\nu}h_m\right)\right] = \frac{C_2}{\lambda^{\nu-1}}, \quad \varkappa = \frac{\nu+2}{\nu}\end{gathered} \tag{2.12}$$

The structure of integral (2.12) is the same as that of integral (2.9): the first two terms corresponding to the linearized energy integral, and the last to the linearized momentum integral [4] multiplied by λ.

Equation (1.12) of divergent form with condition (2.10) yields formula (2.11) which with the use of constant C_3 may be readily written in the form of the first integral

$$\begin{gathered}\left(\frac{\nu+1}{\nu n}\right)^2\lambda^2\left[\lambda g_m - \frac{\nu}{\nu+1}(gf_m + fg_m)\right] - \frac{2(\nu+1)}{\nu n}\lambda\left[\lambda(gf_m + fg_m) - \right. \\ \left. \frac{\nu}{\nu+1}\left(2fgf_m + f^2g_m + \frac{1}{\nu}h_m\right)\right] + \lambda(2fgf_m + f^2g_m + h_m) - \\ \frac{\nu}{\nu+1}\left[\left(3f^2g + \frac{\nu+2}{\nu}h\right)f_m + f^3g_m + \frac{\nu+2}{\nu}fh_m\right] = \frac{C_3}{\lambda^{\nu-1}}\end{gathered} \tag{2.13}$$

$$\varkappa = (\nu + 2) / \nu, \quad m = n(\nu + 2)^2 / 2$$

In conformity with the generating equation (1.12) the integral (2.13) consists of linearized integrals of mass multiplied by λ^2, of momentum multiplied by λ, and of the energy integral [4].

Equations (1.9) and (1.10) can only be used for the derivation of final relationships for functions (2.5) in the case of motions with plane waves. Equation (1.9) generates the momentum integral, and Eq. (1.10) generates for second approximation functions and for any $\varkappa$ the new integral

$$\frac{\varkappa+1}{2n}\lambda\left[\lambda g_m - \frac{2}{\varkappa+1}(gf_m + fg_m)\right] - \left[\lambda(fg_m + gf_m) - \frac{1}{\varkappa+1}(4fgf_m + 2f^2g_m + (\varkappa-1)h_m)\right] = C_4 \qquad (2.14)$$

$$\nu = 1, \quad m = 3n$$

where C_4 is an arbitrary constant. The first term of the integral corresponds to the linearized integral of mass multiplied by λ, and the second to the momentum integral [4].

For cylindrically symmetric shock waves (1.3) ($\nu = 2$) it is, also, possible to use Eq. (1.10). As was done in [4], we set $R_2 = \cos\varphi$ for analyzing the equation of momentum conservation and obtain the following expansion of the unknown functions:

$$v_r = \frac{2n}{\varkappa+1} b^n t^{n-1}[f(\lambda) + t^{-m/2} f_m(\lambda)\cos\varphi + \cdots]$$

$$v_\varphi = \frac{2n}{\varkappa+1} b^n t^{n-1-m/2} u_m(\lambda)\sin\varphi + \cdots$$

$$\rho = \frac{\varkappa+1}{\varkappa-1}\rho_1[g(\lambda) + t^{-m/2} g_m(\lambda)\cos\varphi + \cdots]$$

$$p = \frac{2n^2}{\varkappa+1}\rho_1 b^{2n} t^{2(n-1)}[h(\lambda) + t^{-m/2} h_m(\lambda)\cos\varphi + \cdots]$$

The system of equations which is satisfied by second approximation functions appears in [4]. Using the method indicated above and introducing the arbitrary constant C_5, for the second approximation functions we obtain

$$\frac{\varkappa+1}{2n}\lambda\left[\lambda g_m - \frac{2}{\varkappa+1}(gf_m + fg_m)\right] - \left[\lambda(gf_m + fg_m - gu_m) - \frac{1}{\varkappa+1}(4gff_m + 2f^2g_m - 2gfu_m + (\varkappa-1)h_m)\right] = \frac{C_5}{\lambda}, \qquad m = 6n \qquad (2.15)$$

For spherically symmetric shock waves (2.6) Eq. (1.10) yields in the first approximation an integral, if $R_2 = \cos\vartheta$ is assumed. Expansion of the unknown functions must be written in the form

$$v_r = \frac{2n}{\varkappa+1} b^n t^{n-1}[f(\lambda) + t^{-2m/5} f_m(\lambda)\cos\vartheta + \cdots]$$

$$v_\varphi = \frac{2n}{\varkappa+1} b^n t^{n-1-2m/5} u_m(\lambda) + \cdots$$

$$v_\vartheta = \frac{2n}{\varkappa+1} b^n t^{n-1-2m/5} w_m(\lambda)\sin\vartheta + \cdots$$

$$\rho = \frac{\varkappa+1}{\varkappa-1}\rho_1[g(\lambda) + t^{-2m/5} g_m(\lambda)\cos\vartheta + \cdots]$$

$$p = \frac{2n^2}{\varkappa+1}\rho_1 b^{2n} t^{2(n-1)}[h(\lambda) + t^{-2m/5} h_m(\lambda)\cos\vartheta + \cdots]$$

Introducing the arbitrary constant C_6, we obtain

$$\frac{\varkappa+1}{2n}\lambda\left[\lambda g_m-\frac{2}{\varkappa+1}(gf_m+fg_m)\right]-\left[\lambda(gf_m+fg_m-2gw_m)-\right. \qquad (2.16)$$
$$\left.\frac{1}{\varkappa+1}(4fgf_m+2f^2g_m-4fgw_m+(\varkappa-1)h_m)\right]=\frac{C_6}{\lambda^2} \qquad m=10n$$

The structure of integrals (2.15) and (2.16) is the same as that of integral (2.14), with the first term corresponding to the linearized integral of mass and the second to the linearized momentum integral [4].

Equation (1.9) of the divergent form yields for $\nu = 2$ and $\nu = 3$ the integral which defines flows with conservation of the moment of momentum of flow; such flows cannot be defined by expansions (2.3) for the shock wave propagating in a quiescent gas, and are not considered here.

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FLOWS OF A REACTING MIXTURE IN LAVAL NOZZLES UNDER CONDITIONS OF A QUASI-FROZEN PROCESS

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Flows of a chemically active gas mixture are considered in a small region of a Laval nozzle, where their mode changes from subsonic to supersonic (the frozen speed of sound is considered) are analyzed. Continuous solutions and solutions with shock waves are derived. Conditions of shock-free flows are obtained.